

Topic : Complex Number

Type of Questions	M.M., Min.
Single choice Objective (no negative marking) Q. 1,2,3,4,5,6 (3 marks, 3 min.)	[18, 18]
Multiple choice objective (no negative marking) Q.7, 8 (5 marks, 4 min.)	[10, 8]
Subjective Questions (no negative marking) Q. 9,10,11,12,14 (4 marks, 5 min.)	[20, 25]
Match the Following (no negative marking) Q.13 (8 marks, 8 min.)	[8, 8]

- The number of complex numbers  $z$  such that  $|z - 1| = |z + 1| = |z - i|$  equals  
(A) 1 (B) 2 (C)  $\infty$  (D) 0
- If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - x + 1 = 0$ , then  $\alpha^{2009} + \beta^{2009} =$   
(A) -1 (B) 1 (C) 2 (D) -2
- If  $\omega$  be an imaginary cube root of unity, then the number :  
 $(1 - \omega - \omega^2)^3 + (\omega - 1 - \omega^2)^3 + (\omega^2 - \omega - 1)^3$  is:  
(A) divisible by 3 but not by 8 (B) divisible by 8 but not by 3  
(C) divisible by both 3 & 8 (D) none of these
- If the imaginary part of the expression  $\frac{z-1}{e^{i\theta}} + \frac{e^{i\theta}}{z-1}$  be zero, then the locus of  $z$  is  
(A) a straight line parallel to x-axis (B) a parabola  
(C) a circle of radius 1 (D) a straight line passing through (1, 0)
- The reflection of the complex number  $(2 - i)$  in the straight line  $iz = \bar{z}$  is  
(A)  $4 - 3i$  (B)  $3 + 4i$  (C)  $2 + i$  (D)  $1 - 2i$
- If  $z_1, z_2, z_3, z_4$  are imaginary 5<sup>th</sup> roots of unity, then the value of  $\sum_{r=1}^{16} (z_1^r + z_2^r + z_3^r + z_4^r)$ , is  
(A) 0 (B) -1 (C) 20 (D) 19
- If  $z_1$  and  $z_2$  are two complex numbers satisfying the equation  
 $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$  then  $z_1/z_2$  is a number which is  
(A) positive real (B) negative real (C) imaginary (D) purely imaginary
- The complex number  $z$  satisfying  $|z + \bar{z}| + |z - \bar{z}| = 2$  and  $|iz - 1| + |z - i| = 2$  is/are  
(A)  $i$  (B)  $-i$  (C)  $\frac{1}{i}$  (D)  $\frac{1}{i^3}$



9. Compute the product ,  $\left[1 + \left(\frac{1+i}{2}\right)\right] \left[1 + \left(\frac{1+i}{2}\right)^2\right] \left[1 + \left(\frac{1+i}{2}\right)^{2^2}\right] \dots \dots \left[1 + \left(\frac{1+i}{2}\right)^{2^n}\right]$  where  $n \geq 2$

10. Let A and B be two complex numbers such that  $\frac{A}{B} + \frac{B}{A} = 1$ , then prove that the origin and the two points represented by A and B form vertices of an equilateral triangle.

11. Find the equation of line joining the points  $(1 + i)$  and  $2 - i$  in complex plane.

12. Let  $z_1 = 10 + 6i$  and  $z_2 = 4 + 2i$  be two complex numbers and z be a complex number such that

$\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$ . Find the centre and radius of the locus of complex number z.

13. Match the column :

Column-I

Column-II

- |   |                                   |
|---|-----------------------------------|
| (A) If $\omega_1, \omega_2$ be imaginary cube roots of unity, then $\omega_1^4 + \omega_2^4$ is equal to                      | (p) $-\frac{1}{\omega_1\omega_2}$ |
| (B) If $\omega \neq 1$ be nth roots of unity, then $\omega + \omega^2 + \omega^3 + \dots + \omega^{n-1}$ is equal to          | (q) $-1$                          |
| (C) If $z_1$ and $z_2$ be two nth roots of unity, then $\arg\left(\frac{z_1}{z_2}\right)$ is a multiple of                    | (r) $\frac{2\pi}{n}$              |
| (D) If $\omega \neq 1$ be nth roots of unity, then value of $(1 - \omega)(1 - \omega^2) \dots (1 - \omega^{n-1})$ is equal to | (s) $n$                           |

14. Draw the locus of z :

- (i)  $\arg(z - 1 + i) \leq -\frac{\pi}{3}$
- (ii)  $|z + 1 - i| = |z - 2|$
- (iii)  $|z| \leq 1$  and  $-\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$
- (iv)  $\arg\left(\frac{z+i}{z-i}\right) = \frac{2\pi}{3}$



# Answers Key

1. (A)    2. (B)    3. (C)    4. (C)

5. (D)    6. (B)    7. (C)(D)

8. (A)(B)(C)(D)    9.  $\left(1 - \frac{1}{2^{2^n}}\right) (1 + i)$

11.  $z(1+2i) - \bar{z}(1-2i) - 6i = 0$

12. centre:  $9 + i$ , radius =  $\sqrt{26}$

13. (A)  $\rightarrow (p,q)$ , (B)  $\rightarrow (p,q)$ , (C)  $\rightarrow (r)$ , (D)  $\rightarrow (s)$

